Report 3

Diffuser

Written by

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Abstract

This document is part of a series of reports covering the topic "Room Acoustics", which was conducted as a self study course with kind assistance from Jonas Brunskog from the Danish Technical University.

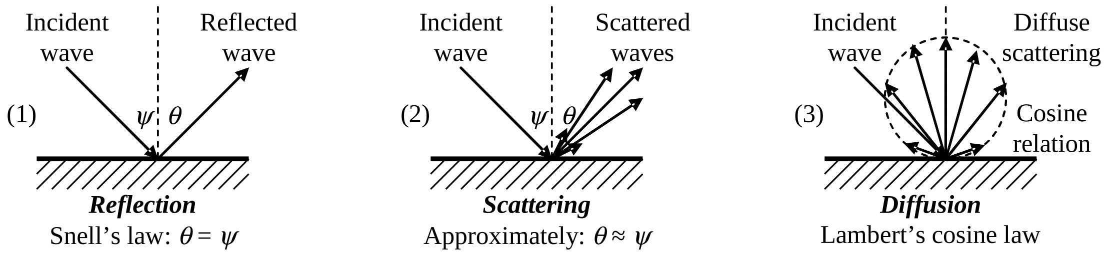
The objective is diffuser design, which initiated a study of scattering defining the relation to panel size and wavelength and also with some notes on diffraction. The study introduces the design of the popular Schröder quadratic residue diffuser and the benefits and consequences of making the diffuser periodic.

1. Sound diffusion

Summary – In physics diffusion is the random spread of matter from high to low concentration, and in this context the term is used to describe the scattering of sound when it hits an object; and here especially the Schröder diffuser. Sound scattering contradicts the conventional view that sound is reflected just as light rays are reflected within a mirror. The theory behind sound scattering is not straight forward and useful explanations were only found as bits and pieces within the available literature so this report is introduced by assembling notes on the nature of scattering.

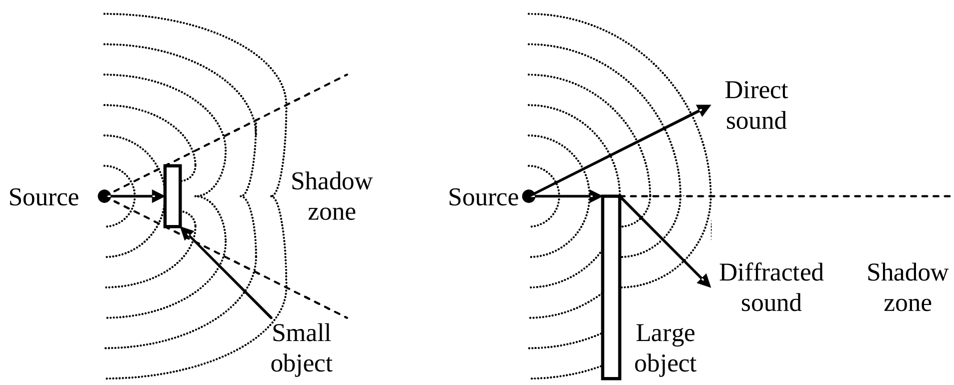
1.1. Questions

Within the geometrical room acoustics sound waves are assumed to follow the direct path with reflection at walls and panels according to Snell's law. However, sound waves are scattered into directions which does not obey Snell's law, and within theoretical room acoustics is scattering represented by Lambert's cosine law where the angle of incidence is unimportant. Both relations cannot be true so what is the appropriate description?



(1) Sound waves are reflected within a large surface just as light rays hitting a mirror so the angle of the incident sound is equal to the angle of the reflected sound (Snell's law used within acoustics).
(2) Sound waves are scattered randomly although the main energy is reflected according to Snell.
(3) Sound waves are scattered randomly into any direction according to Lambert's law.

The incident energy is proportional to area assuming plane waves and normal incidence so panels can be expected to scatter the energy in proportion to area, but scattering is also related to frequency with most energy being reflected at high frequencies. A physically *large* object can thus be *small* from an acoustical point of view, so what is the relation to panel dimension and frequency?



Sound waves are not blocked by an object but are bending around the object. The sudden change in propagation impedance generates reflections from the boundary, *edge diffraction* and this causes interference with the direct signal at the receiving point.

Sound can be heard although an object is blocking the line of sight so sound can bend around the boundaries of an object and radiate into the "shadow zone" behind the object while light is blocked and casts a shadow (to a first approximation at least). The mechanism is called *diffraction* and often without further explanation; but what are the consequences?

1.2. Answers

The questions are attempted answered below. The treatment is not rigorously but a group of results assembled in order to gain some insight.

1.2.1. Scattering

Starting with an infinitely large panel with uniform impedance, the sound pressure at the surface is given by the reflection coefficient R and the sound pressure p_i of the incident wave [K-38].

$$p_s = (1+R)\,p_i$$

For a completely reflecting surface we have $R = 1$, so the reflected sound is of the same amplitude and phase as the incident wave and the sound pressure at the surface becomes twice the incident pressure assuming plane waves and normal incidence of the sound wave. This equation applies to the plane waves propagating within Kundt's tube for impedance measurements and at least to a first approximation for waves propagating within a room and hitting a wall. For the use with panels of limited size the assumption is made that the panel is sufficiently large so that the pressure cannot be levelled with the pressure at the remote side [CA-296], which is to say that there is no significant edge diffraction; hence, the boundary is located far away and the panel is acoustically *large*.

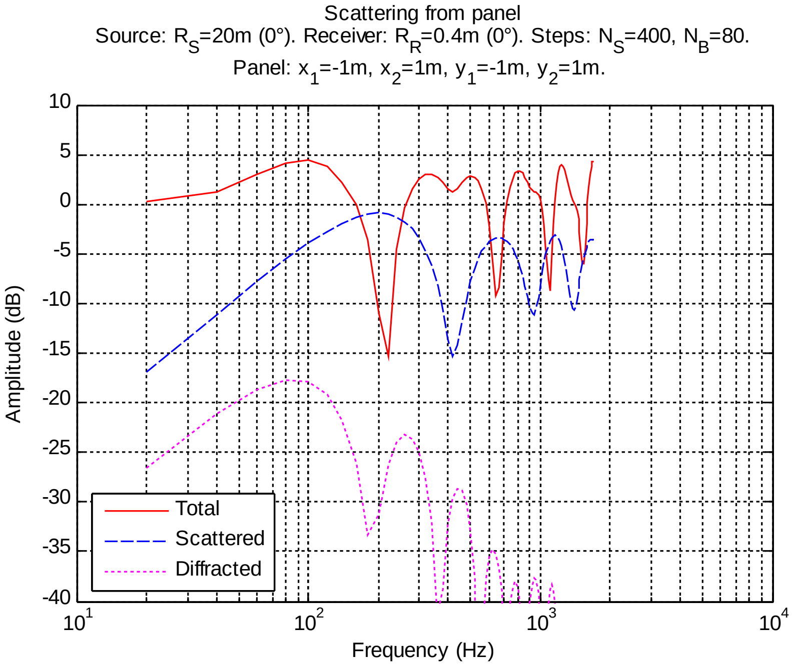
A description for *small* panels appears to be solving the Helmholz-Kirchhoff integral equation with proper boundary conditions, which generally requires the use of boundary element methods for numerical solving of the partial differential equations [CA-252]. Numerical solution is a powerful tool but an analytic relation between the sound pressure and the parameters would clearly indicate proportionalities. An analytic solution exists for the far field scattering from a rectangular panel with the measures $2a$ and $2b$, the source located at $r_0 = (x_0, y_0, z_0)$ and the receiver at $r = (x, y, z)$, or vice versa, and θ for the reflection angle [CA-274].

$$\underbrace{p_S(\mathbf{r})}_{\substack{\text{Scattered}\\ \text{sound}\\ \text{pressure}}} \propto \underbrace{\frac{ikab}{2\pi^2 \mathbf{r}\mathbf{r_0}}}_{\substack{\text{Relations to:}\\ \text{Frequency }(k)\\ \text{Size (area, }ab)\\ \text{Distance }(\mathbf{r},\,\mathbf{r}_0)}} \underbrace{\exp(-ik(\mathbf{r}+\mathbf{r_0}))}_{\substack{\text{Phase due}\\ \text{to travelled}\\ \text{distance}}} \underbrace{\operatorname{sinc}\left(ka\left(\frac{x}{|\mathbf{r}|}+\frac{x_0}{|\mathbf{r_0}|}\right)\right)\operatorname{sinc}\left(kb\left(\frac{y}{|\mathbf{r}|}+\frac{y_0}{|\mathbf{r_0}|}\right)\right)}_{\substack{\text{Amplitude due to}\\ \text{offsetting source}\\ \text{and receiver}}} \underbrace{\cos(\theta)}_{\substack{\text{Angle to the}\\ \text{receiver}}}$$

The equation shows that the amplitude of the scattered signal is proportional to frequency due to the angular wave number $k = 2\pi f/c$, and it is also proportional to area through the product ab. The relation to receiving angle θ indicates diffuse reflection according to Lambert's cosine law, and the location vectors indicate an inverse distance law, which is intuitively understandable but not easily proved since it involves very large distances [CA-124].

Although on fairly safe ground, the above expression does not give an explanation of why the sound is scattered in different directions or why the relation is proportional to frequency, so an analysis was carried out in section 3.1 trying out some theory. The base of the theory is that sound pressure is a scalar and as such independent upon direction. Using Huygens principle the surface is divided into small sections radiating into space as consequence of the incident pressure wave, and the sound pressure at the receiver becomes the superposition of these wavelets. This method does not assume anything about the size of the panel compared to wave length; however, the theory does indicate proportionality to frequency through the use of point sources representing the wavelets; the sound pressure from a point source is strictly frequency proportional.

The result is shown below for the source within far field and the receiver within near field. The red line is the resultant sound pressure, which for an infinite panel equals twice the incident value (see section 3.1.4) but the result is lower since the scattered sound (dashed blue) is proportional to frequency and not at full level. The receiver is offset 0.4 m so the scattered sound is delayed and destructive interference occurs at 220 Hz where two times the distance equals half wave length. The diffracted signal from the edge of the panel is of insignificant amplitude and does not cause trouble.



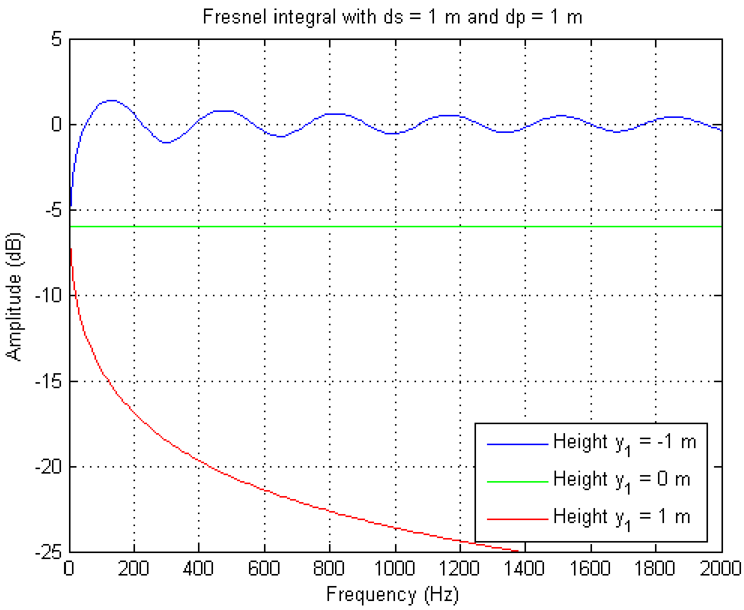
The response of the scattered sound for the source at 20 m and the receiver 400 mm from the panel with the measure 2 m by 2 m.

The scattered signal is proportional to frequency below 200 Hz but ripples at higher frequencies thus not following the analytic expression. The numerical tool indicates relation to the distance to source and receiver and Cox and D'Antonio quotes Rindel for a response approaching that of a high pass filter [CA-332]. This initiated the study presented in section 3.2, which resulted in a formula for estimation of the cut off frequency of the scattered sound.

The relation to observation angle was much more pronounced than expected with a lobe at 0º and the level of scattering was reduced much faster than according to the cosine relation (–20 dB at ±60º versus –6 dB for the cosine; see section 3.1.4). With an option selected for inclusion of a cosine relation similar to Lambert's law the sound pressure close to the surface was reduced toward zero where doubling is the correct answer; this disproves the cosine in the analytic expression.

1.2.2. Edge diffraction

Objects are acoustically *large* or *small* when related to wave length, so an object may be large at high frequencies and small at low frequencies. A large object scatters the incident sound efficiently so the sound pressure is expected to approach zero at the remote side of the object; however, this is not the case in real life. An example is a large building separating a noisy road from a silent garden. The building is expected to block high frequencies from entering the garden but the sound being heard is not only the low-frequency rumbling; the higher frequencies are audible too so the sound manages some how to surpass the building. This is due to diffraction and this issue is lightly treated within sections 3.3 to 3.5.



The level of the signal at the receiver is shown for a semi-infinite baffle blocking the lower part of the propagating wave front. The direct signal is interrupted for the lower curve (red colour).

Using Fresnel diffraction theory the sound pressure of the diffracted signal within the shadow zone can be shown to be inversely proportional to the square root of frequency, i.e. the slope approaches the value –3 dB/octave (see section 3.5), so the sound can be represented by the treble control on an amplifier when the knob is turned down.

An interesting point to note is the subjective impression of the sound attenuation due to the building blocking the sound from the road. This is perceived as a reduction of the overall sound level and not just a high frequency loss. This is due to the low-frequency compression characteristic of the hearing system at moderately high sound pressure levels thus creating the impression of attenuation of low frequencies even though the loudness is reduced only moderately. In short, high frequencies are reduced by the obstruction and low frequencies by our sensory system so the overall impression is close to that of a level reduction and not the expected signal colouring due to the slope.

1.2.3. Conclusion

The scattered sound pressure is proportional to area and inversely proportional to distance for the source within the far field. The incident pressure wave front is re-radiated with a preferred direction given by Snell’s law although with some beam width, so scattering is neither oblique reflection nor completely diffuse and the beam width is inversely related to the panel size. The proportionality to frequency is valid for low frequencies but is lost above a cut off frequency due to interference when source or receiver is within the near field. A pressure wave propagates along the surface toward the boundary creating diffraction but the interference is minimal. The signal is capable of bending around an obstacle and the consequence is a reduction within the high frequency range. Increasing the size of the panel reduces the cut off frequency but the slope remains unchanged so the object must be enormous to significantly reduce the overall level.

2. Schröder diffuser design

Summary – The design of a Schröder diffuser is presented and the consequence of arranging several diffusers into a periodic setup is studied.

2.1. Design method

The method proposed by Schröder was to implement acoustical delay lines and adding the delayed replicas of the incident sound wave into a pattern defined through interference. The delay lines are formed from channels with different lengths and rigid termination at the opposite end of the channel. The incident wave travels within the channel with reflection at the rigid end so the delay is equal to two times the channel length. The length of channel is defined through number theory by the value s_n and the design parameter the incremental channel length Δd.

$$\tau_n = \frac{2d_n}{c}, \quad d_n = s_n \Delta d$$

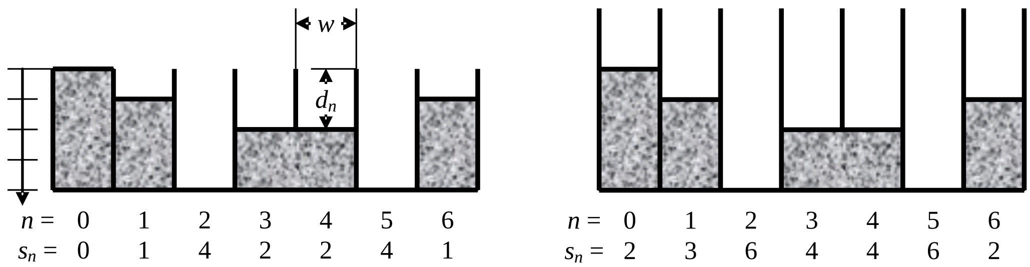
When the reflected signal is retransmitted into the space in front of the diffuser partial reflection takes place, which may cause standing waves (resonances) within the channels; however this issue will be ignored. The channels are narrow so the signal is scattered within the space in front of the diffuser (see section 2.2). Furthermore, the channels are assumed not to interact (locally reacting).

2.1.1. Quadratic residue sequence

A common algorithm is the quadratic residue sequence [CA-291]. The input is $n = 0, 1, 2 \ldots N - 1$, where N is the sequence length and the output is the remainder when N has been subtracted the required number of times to move the value of s_n below N. The sequence length must be prime to avoid repetitions (for instance $N = 4$ gives $s_n = 0\ 1\ 0\ 1$).

$$s_n = n^2 \bmod N, \quad s_n = \begin{cases} N = 5: & 0 \quad 1 \quad 4 \quad 4 \quad 1 \\ N = 7: & 0 \quad 1 \quad 4 \quad 2 \quad 2 \quad 4 \quad 1 \end{cases}$$

The quadratic residue diffuser is shown below for $N = 7$ (left picture).



Cross section through a QRD (Quadratic Residue Diffuser) with length N = 7 (left) and the same design with improved low frequency operation (right).

Low frequency operation is improved by arranging for the largest sequence value $s_{max} = N - 1$ since this produces the longest channel. The sequence is modified adding offset m and using $m = 1$ gives $s_{max} = 5$, which is an improvement, while $m = 2$ gives $s_{max} = 6$, which is the highest possible value with $N = 7$ and results in the diffuser design shown above (right picture).

$$s_n = \left(n^2 + m\right) \bmod N, \quad s_n = \begin{cases} N = 7 \quad m = 1: & 1 \quad 2 \quad 5 \quad 3 \quad 3 \quad 5 \quad 2 \\ N = 7 \quad m = 2: & 2 \quad 3 \quad 6 \quad 4 \quad 4 \quad 6 \quad 3 \end{cases}$$

2.1.2. Primitive root sequence

Another algorithm uses the primitive root r for the odd prime N [CA-298]. The algorithm generates the entire range of integer values from 1 to $N - 1$ for $n = 1, 2, \dots N - 1$ in contrast to the quadratic residue algorithm so optimising is not required. The table below is from Wikipedia.

$$\begin{array}{l} s_n = r^n \bmod N \\ s_n = \left(r \cdot s_{n-1}\right) \bmod N \end{array}, \quad \text{where} \quad \begin{cases} N = & 3 & 5 & 7 & 11 & 13 & 17 & 19 & 23 & 27 & 29 \\ r = & 2 & 2 & 3 & 2 & 6 & 10 & 10 & 10 & 2 & 10 \end{cases}$$

The first value is the root itself and $N = 7$ gives $s_n = 3, 2, 6, 4, 5, 1$. The sequence is asymmetrical so two sequences can be appended with one reversed to build a larger diffuser using smaller units. This is in opposition to the quadratic residue sequence which is symmetrical [CA-311].

2.1.3. Design equations

<u>High frequency</u> operation is limited by the assumption of plane waves propagating through the channels since the channel width w must be less than wavelength in order to suppress cross modes; with recommended half wave length criteria [CA-291]. The useful frequency range is not restricted to the calculated limit but the theory does not apply at higher frequencies since cross modes cannot be avoided. The recommended lowest value is 25 mm to keep viscous absorption low [CA-294]; in this case the highest design target is 7 kHz although diffusion is obtained at higher frequencies.

$$w < \frac{\lambda_{\min}}{2} = \frac{c}{2 f_{\max}} \quad \Rightarrow \quad f_{\max} < \frac{c}{2w}$$

<u>Low frequency</u> operation is defined by the longest channel within the diffuser. Since the maximum sequence value is limited to below N, the lowest frequency is reached when $s_{max} = N - 1$ and the sequence could be arranged to reach this value. With an increment of $\Delta d = 50$ mm a sequence with $N = 7$ can operate down to approximately 570 Hz.

$$f_{\min} = \frac{1}{\tau_{\max}} = \frac{c}{2d_{\max}} = \frac{c}{2s_{\max}\Delta d} \quad \Rightarrow \quad f_{\min} \geq \frac{c}{2(N-1)\Delta d}$$

The lower frequency is not a steep cut off; the performance degrades at lower frequencies so a *design frequency* is defined using the sequence length[1]. For $N = 7$ the design frequency becomes 490 Hz assuming $\Delta d = 50$ mm.

$$f_0 = \frac{c}{2N\Delta d}$$

<u>Critical frequencies</u> are encountered when all channels radiate in phase and the diffuser operates as a rigid surface with strong reflection. This situation occur at frequencies where Δd corresponds to integer multiple of half wave lengths [CA-295].

$$\Delta d = \frac{m\lambda_C}{2} = \frac{mc}{2 f_C} \quad \Rightarrow \quad f_C = \frac{mc}{2\Delta d} = mNf_0, \quad m = 1, 2, 3, \dots$$

[1] Cox and D'Antonio use another definition within the text book [CA-294]. However, their equation can be reduced to the above equation if d_{max} is substituted by $s_{max}\Delta d$:

$$f_0 = \frac{s_{\max}}{N}\frac{c}{2d_{\max}} = \frac{s_{\max}}{N}\frac{c}{2s_{\max}\Delta d} = \frac{c}{2N\Delta d}$$

To avoid the critical frequencies within the design bandwidth the first critical frequency is moved above the upper frequency limit. This defines a limiting value of the incremental length.

$$f_C > f_{\max} \quad \Rightarrow \quad \frac{c}{2\Delta d} > \frac{c}{2w} \quad \Rightarrow \quad \Delta d < w$$

The requirement may also propose a minimum sequence length [CA-295].

$$f_C = \frac{c}{2\Delta d} = Nf_0 \quad \wedge \quad f_C > f_{\max} \quad \Rightarrow \quad N > \frac{c}{2wf_0}$$

2.1.4. Design example

A design proposal will be given for a diffuser covering the range from 500 Hz to 5 kHz. Operation is not limited to this range; the diffuser is known to produce more scattering than a plane surface for one to two octaves below the design frequency [CA-292]. Operation above the maximum frequency is limited by the critical frequencies where the diffuser behaves as a rigid surface.

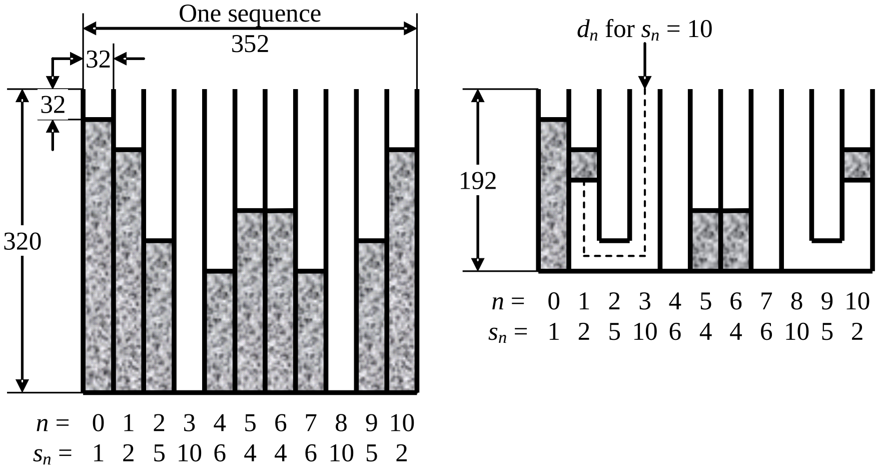
1. The channel width is calculated from $w < c/2f_{max}$ to maximum 34 mm and w = 32 mm was selected although close to the lowest recommended channel width of 25 mm, so the diffuser may introduce some absorption due to boundary layer effects.
2. The channel increment is set to the same value $\Delta d = w$ = 32 mm thus moving the critical frequencies outside the intended range.
3. For a design frequency of 500 Hz the sequence length becomes $N > c/2wf_0$ with value 10.7 so the nearest higher prime number is selected to N = 11.
4. Check the resulting design: $f_0 = c/2N\Delta d$ = 490 Hz and $f_{\max} = c/2w$ = 5.4 kHz.
5. The sequence is then build (here using the quadratic residue sequence) and offset searching for a sequence where $s_{max} = N - 1 = 10$; this is obtained for m = 1 (shown in bold face).

M	$s_n = (n^2 + m) \bmod 11$											s_{max}
	0	1	2	3	4	5	6	7	8	9	10	
0	0	1	4	9	5	3	3	5	9	4	1	9
1	**1**	**2**	**5**	**10**	**6**	**4**	**4**	**6**	**10**	**5**	**2**	10

6. The channel lengths are calculated from $d_n = s_n\Delta d$ as follows: d_0 = 32 mm, $d_1 = d_{10}$ = 64 mm, $d_2 = d_9$ = 160 mm, $d_3 = d_8$ = 320 mm, $d_4 = d_7$ = 192 mm and $d_5 = d_6$ = 128 mm,

The design is shown below as a cross section (side view or top view). The total depth of the diffuser is $s_{max}\Delta d$ = 320 mm and the width is $N\Delta d$ = 352 mm. A practical design must allow for material thickness and using 4 mm plywood the overall width increases with 12·4 mm = 48 mm to 400 mm for one single period and 11·4 mm = 44 mm for each section if the diffuser is periodic (plus 4 mm for the last section).

Lots of space is wasted by the blocked parts of the channels but the depth can be reduced if the long channels with s_n = 10 is folded; the depth is reduced to 192 mm, a saving of 40 % in diffuser depth. The bended channels should preferably be non-integer lengths for the initial part to reduce the problem at the critical frequencies where unbend channels are integer multiples of half wave lengths [CA-313]; however, this is not realised by the present example.



Cross section through a QRD with three sections each with length *N* = 11 (left) and modified for material saving and depth reduction (right). Dimensions are in millimetres.

2.2. Notes

The Schröder theory was developed using the Fraunhofer far field theory thus assuming that the sound waves can be approximated by plane waves. This does not apply to the usual implementation where source and receiver are close to the diffuser but theory and measurements are not that far from each other according to Cox and D’Antonio [CA-322].

The channels are assumed to radiate into space as if the radiation came from a narrow slit with zero width. However, the channel width w is significant at high frequencies so signal spreading is limited to $kw < 2$ for an observation angle of 90° i.e. along the surface of the diffuser; this corresponds to a limiting frequency of 2.4 kHz for $w = 32$ mm where the amplitude is reduced 3 dB, or 11 kHz at the frequency of the first null [S-103].

$$f_{-3\mathrm{dB}} = \frac{0.22c}{w\sin(\theta)}, \quad f_{null} = \frac{c}{w\sin(\theta)}$$

The diffuser described so far covers less than 1 m^2 of surface so the scattered energy is negligible for almost any application and it is very inefficient at low frequencies. The minimum frequency for a narrow panel is calculated from the equation below (see section 3.2)

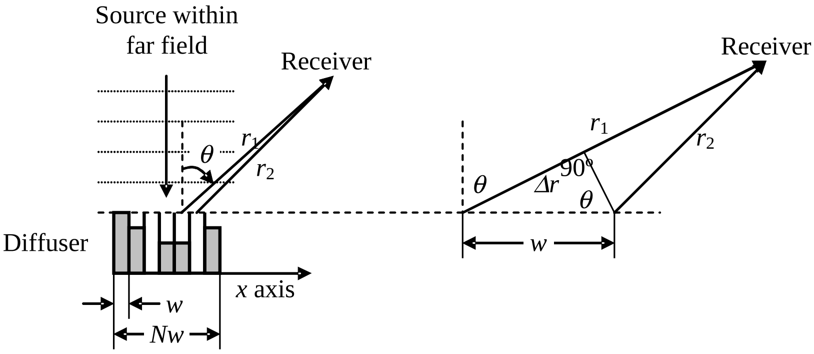
$$f_{\mathrm{cut\,off}} \approx \frac{c\hat{r}}{2h^2}, \quad \hat{r} = \frac{2r_S r_R}{r_S + r_R}$$

The expression defines the lowest useful frequency below which the efficiency is proportional to frequency as was discussed within the introduction. The cut off frequency is around 5.4 kHz using panel width $h = 0.4$ m and an arbitrary distance to source and receiver of $r_S = r_R = 5$ m, so operation at the design frequency of 500 Hz results in 20 dB attenuation of the lowest frequencies.

In order to improve low-frequency operation the diffuser must be assembled from several sections and using the above estimation about 4 sections is the minimum number for operation down to the design frequency. The resulting width of 1.6 m may violate the assumption of far field operation but this issue will be ignored. What is more important is that the diffuser becomes periodic so the sound pressure at the receiving location is given through interference of the scattered sound from several sections. This issue will be studied in more detail in the following sections, which start off by analysing the polar response of the diffuser and then the consequence of using several sections into a periodic diffuser.

2.3. Scattering

The level at the receiver position is defined through interference between the scattered components from the diffuser and depends upon the observation angle and the frequency. The response of the diffuser expressed as amplitude versus observation angle can be expected to depend upon frequency and vice versa, so it is not possible to design a diffuser with even scattering and flat amplitude.



The scattering from the diffuser is calculated through interference.

The wave being reflected from the diffuser undergoes changes in amplitude and phase due to the reflection coefficient R_n and the phase is also altered due to the distance to the receiver but the relation of amplitude with distance will be ignored. The contribution from channel number n is expressed by the transfer function H_n, which relates only to the reflection coefficient and the delay.

$$H_n = R_n \exp(-i2\pi f \tau_n)$$

The distance to the receiver from a given channel is unknown but the change of distance to the next channel is given by the distance between the channels and the observation angle, which is assumed within the far field. For negative observation angle the increment is negative (as within the figure).

$$r_2 = r_1 + w\sin(\theta) \quad \Rightarrow \quad \Delta r = r_2 - r_1 = w\sin(\theta)$$

The delay from the first channel to channel number n becomes:

$$\tau_n = \frac{n\Delta r}{c} = \frac{nw\sin(\theta)}{c}$$

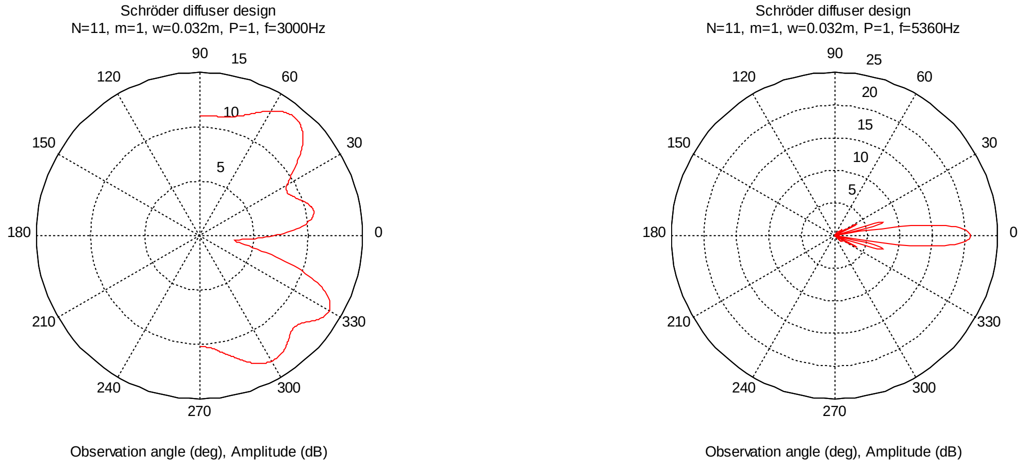
The reflection coefficient for a channel with depth d_n is given by the impedance of the channel, which is defined within the *Absorption* report [R2] for an air cushion of thickness d_n.

$$\left.\begin{aligned} Z_n &= -i\rho_0 c \cot(kd_n) \\ R_n &= \frac{Z_n - \rho_0 c}{Z_n + \rho_0 c} \end{aligned}\right\} \quad \Rightarrow \quad R_n = -\frac{1 + i\cot(kd_n)}{1 - i\cot(kd_n)}$$

The combined action is the reflection coefficient times the expression of the delay and the resulting transfer function is given by adding the contributions.

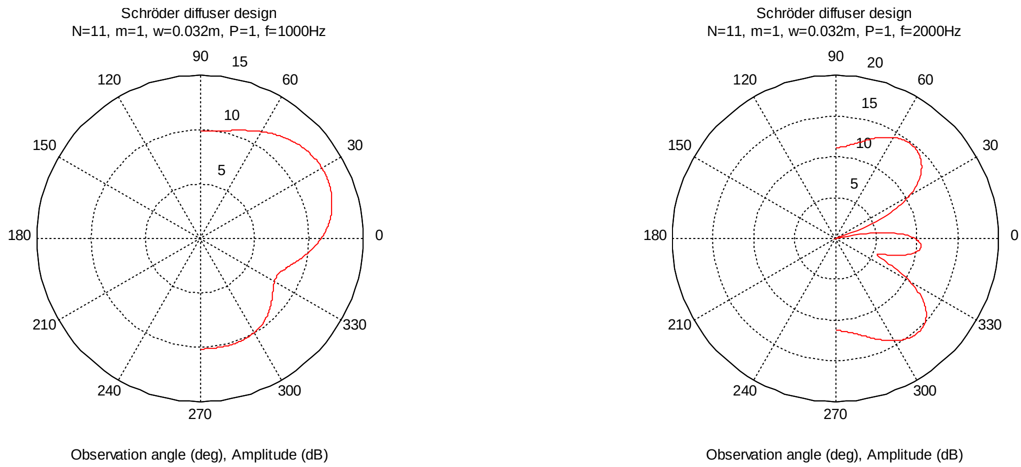
$$H = \sum_{n=1}^{N} H_n = \sum_{n=1}^{N} R_n \exp\left(-\frac{i2\pi f n w \sin(\theta)}{c}\right)$$

The result is shown below for the diffuser from the design example (N = 11) for a frequency within the intended frequency range. The plot to the right is at the critical frequency where the diffuser behaves as a rigid plate; the same plot results when $R_n = 1$ is used. The exponential is unity and the sum becomes N corresponding to an amplitude of $20 \cdot \log_{10}(11) = 20.8$ dB at the peak.



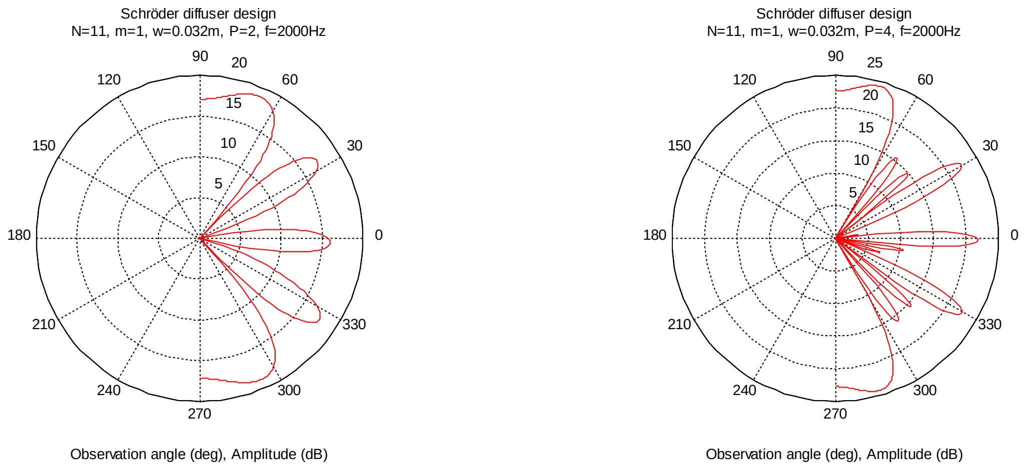
To the left is shown the scattering pattern when operating within the intended frequency range, and to the right when operated right at the critical frequency corresponding to a plane and rigid plate.

Two more plots show the change of patterns at other frequencies.



The scattering patterns at two frequencies within the intended frequency range.

When several identical diffusers are arranged in a sequence the scattering is changed into lobes with beam width inversely related to the panel size.



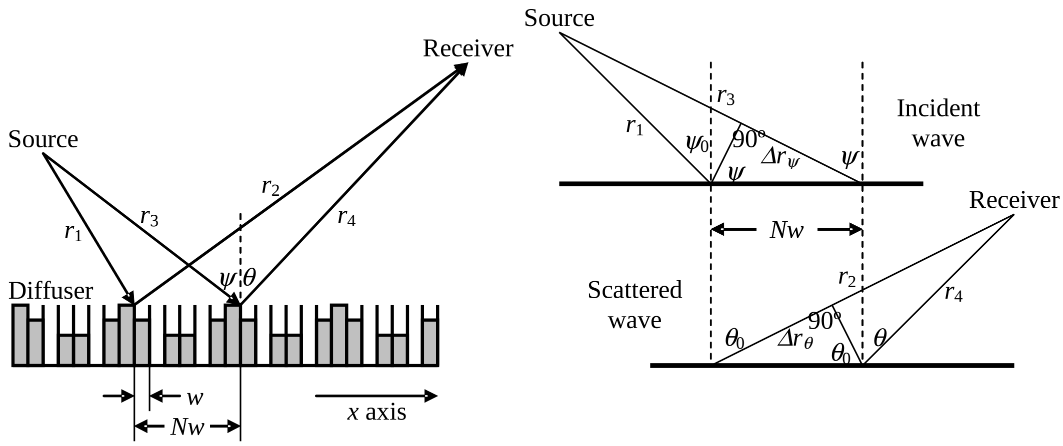
Using several periods of the diffuser results in constructive interference at 0° observation angle and side lobes. The location of the lobes is given by the diffuser width *Nw* as shown in section 2.4.

2.4. Periodic diffuser

An expression of the far field scattering from a diffuser is reproduced below where A is a constant (with dimension of pressure) and $R(x)$ is the reflection coefficient along the surface represented by S and with the variation restricted to the x-axis. The angle of incidence and of scattering is ψ and θ respectively [CA-296].

$$p_S = \left| A \int_S R(x) \exp\left(ikx\left[\sin(\theta) + \sin(\psi)\right]\right) dx \right|$$

An example with a periodic diffuser is shown below illustrating the interference due to different signal paths.



The scattering from different sections of a surface cause interference at the receiver. To the right is shown details of incident and scattered waves in order to calculate the difference in path lengths.

The signal from the source to the receiver may follow different paths; one through $r_1 + r_2$ and another through $r_3 + r_4$, and interference occur at the summing point due to differences within the phase shifts. The interference is constructive when the difference in path lengths corresponds to integer number of wave lengths.

$$\left| r_1 + r_2 - (r_3 + r_4) \right| = m\lambda, \quad m = 0, 1, 2, \ldots$$

The length of the path through r_1 is increased when the incident angle is changed from ψ_0 to ψ with Δr_ψ into $r_3 = r_1 + \Delta r_\psi$ corresponding to the change in distance of Nw along the x axis. The length of r_2 is decreased when the scattered angle is changed from ψ_0 to ψ with Δr_θ into $r_4 = r_2 - \Delta r_\theta$. The right triangles with side length Nw defines the increments; the angle of the scattered signal is in the negative (clockwise) direction according to the normal to the surface; and the receiver is assumed within the far field thus allowing for a simplification ($\theta_0 \approx \theta$).

$$\begin{aligned} r_1 - r_3 &= -\Delta r_\psi \\ r_2 - r_4 &= \Delta r_\theta \end{aligned} \quad \text{where} \quad \begin{aligned} \Delta r_\psi &= Nw\sin(\psi) \\ \Delta r_\theta &= Nw\sin(-\theta_0) \approx -Nw\sin(\theta) \end{aligned}$$

The requirement for the transformed variable for constructive interference at the receiver is thus determined through insertion of the increments into the previous expression [CA-297].

$$\sin(\theta) + \sin(\psi) = \frac{m\lambda}{Nw}, \quad m = 0, \pm 1, \pm 2, \ldots$$

The sum of two sine functions is limited to less than two so the parameter values cannot be selected freely[2]. For an upper limit of unity, thus leaving some space, the limit becomes $m \le 5$ using the diffuser described within section 2.1.4 ($N = 11$, $w = 32$ mm, and $f_{max} = 5.4$ kHz).

$$\left|\frac{m\lambda}{Nw}\right| < 1 \quad \Rightarrow \quad \frac{|m|c}{Nwf_{max}} \quad \Rightarrow \quad |m| < \frac{Nwf_{max}}{c}$$

The sound pressure of the scattering signal at the receiver location becomes (using $k = 2\pi/\lambda$).

$$p_S = \left| A\int_S R(x)\exp\left(ikx\frac{m\lambda}{Nw}\right)dx \right| \quad \Rightarrow \quad p_S = \left| A\int_S R(x)\exp\left(\frac{i2\pi m}{Nw}x\right)dx \right|$$

Assuming that the radiation originates from the middle of the channel; $x = nw$ with the first channel at zero. This allows the integral to be written as a sum where parameter m is the number of the lobe starting with $m = 0$ for the lobe along the direction of reflection according to Snell's law.

$$p_m = \left| A\sum_{n=1}^{N} R_n \exp\left(\frac{i2\pi mn}{N}\right) \right|$$

The exponential is unity for $m = 0, \pm N, \pm 2N$ and higher so the sum reduces to the addition of the reflection coefficients. With a rigid panel there is total reflection ($R_n = 1$) so the sum is equal to N, and this is also the case with a diffuser at the critical frequency where the depth of the channel is an integer times half wave length. At all other lobes the sum is less than N so the energy is lower.

The angle for the lobes for normal incidence can be determined from:

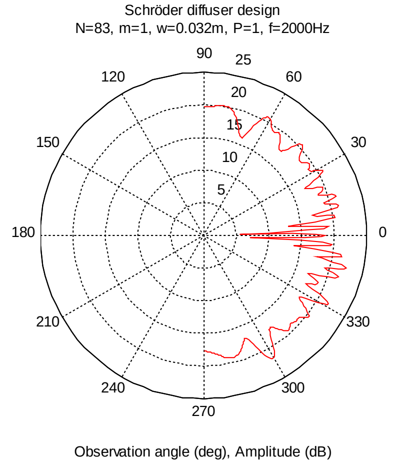
$$\theta = \arcsin\left(\frac{m\lambda}{Nw}\right) = \arcsin\left(\frac{mc}{Nwf}\right), \quad m = 0, \pm 1, \pm 2, \ldots$$

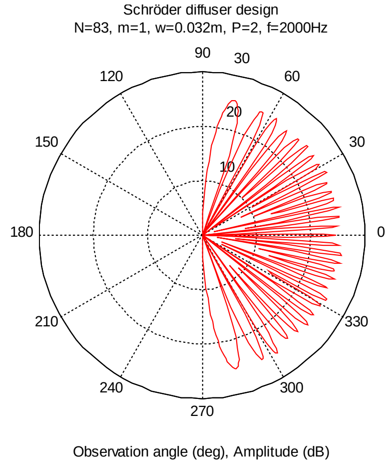
For $N = 11$, $w = 32$ mm and $f = 2$ kHz the lobes occur at 0º, ±29º and ±77º for $m = 0$, 1 and 2; there are no lobes associated with $m > 2$ since the argument to arc sine exceeds unity. The lobes are visible within the last two figures of section 2.2 (at least they come fairly close).

[2] This problem was not addressed by Cox and D'Antonio.

In order to get more lobes, the factor within the denominator *Nw* must be increased so the sequence s_n must be longer. An example is shown below with the frequency selected to 2 kHz for comparison to the pictures within section 2.2. The channels are of the same width as before (w = 32 mm) but the sequence length has been increased to N = 83, which gives a total diffuser width of Nw = 2.7 m. With this setup the lopes appear at 0º, ±3.7º, ±4.7º, ±11.2º and so forth to ±75.6º at m = 15 where the last lobe is found. The lobes are visible only for the periodic diffuser shown right.

The increase in overall level of the periodic diffuser is due to the larger surface area; the diffuser width is doubled and so is the incident energy arriving at the diffuser since plane incident waves are assumed. Two times area means two times the resultant sound pressure at the receiver so the peak level within the right figure will be 6 dB above the left figure.





To the left is shown one period (83 wells) and to the right is shown two periods (166 wells).

In a real setup the source will be within the near field thus introducing a phase profile across the diffuser as well as an amplitude profile, so the estimated diffusion will not be reached; this is not the same as saying that the diffusion is worse with a spherical source within the near field. It should also be noted that the diffusion is dependent upon frequency and the distance from the diffuser to source and receiver; and of course dependent by the design of the diffuser.

In general, the scattering is wildly dependent upon frequency, which may be the reason for the recommendation of the listener not to be located too close to the diffuser.

2.5. Applications

Diffusers find their use in a range of applications:

1. Randomise the initial reflections within a concert hall in order to improve the performance conditions for the musicians and also to direct early reflections toward the audience without creating echoes from panels and side walls. The diffusers can be hung above the scene as transparent panels or placed behind the musicians as curved panels or the more complex structures as the Schröder diffuser.

2. Scattering of sound where an echo would otherwise occur, such as the rear wall within an auditorium, or to suppress flutter echoes between parallel side walls.

3. Sound diffusion within a reverberation chamber.

4. Reduction of reverberation time within long and narrow constructions such as underground railway stations. The diffuser reduces the mean time between reflections thus increasing the number of reflections per second, which again increases sound absorption.
5. Diffusers may be combined with sound absorption when moderate absorption is required; for instance using the wasted space as Helmholtz resonators for low-frequency absorption, or the diffuser can deliberately be designed with boundary layer losses to increase the sound absorption.

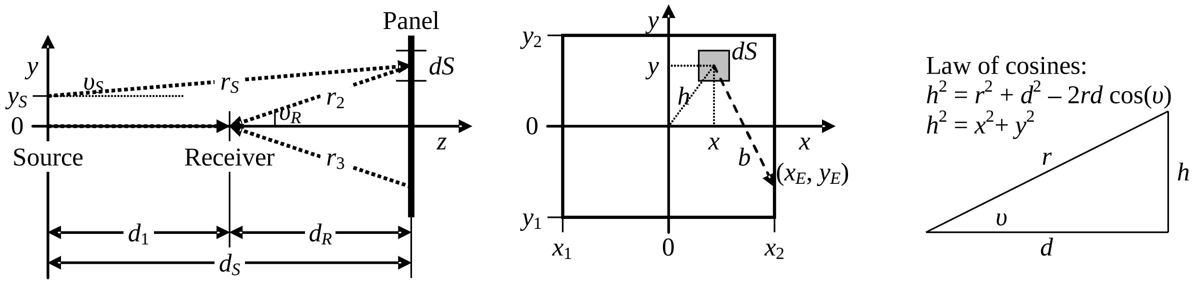
This ends the report for the design and analysis of the quadratic residue diffuser; the remaining of the report is devoted the theory quoted within the initial part of determining a basis for the understanding of scattering.

3. Appendix

This chapter contains additional material referenced within the main text.

3.1. Scattering from panel

The scattering of the sound in front of a rigid panel is analysed to determine the relation between the far field sound pressure and the surface area and frequency. The source and receiver are located at (x_S, y_S, d_S) and (x_R, y_R, d_R). The signal at the receiver comes (1) directly from the source; (2) from the scattered sound from the surface; and (3) from the boundary edge diffraction.



The calculation of distances uses Pythagoras and the calculation of the angle of incidence uses the cosine law.

3.1.1. Direct sound

The sound pressure p_1 at the receiver is given by the direct sound from the point source. The source strength is q (volume velocity) and distance d_1 is directly from source to receiver [S-23].

$$p_1 = \frac{ik\rho_0 c}{4\pi d_1}\exp(-ikd_1)\,q, \quad d_1 = \sqrt{(x_S - x_R)^2 + (y_S - y_R)^2 + (d_S - d_R)^2}$$

3.1.2. Scattered sound

The sound pressure p_2 at the receiver is determined using an approach inspired by the wavelet theory and starts off by defining the sound pressure at point (x, y) on the surface.

$$p_S = \frac{ik\rho_0 c}{4\pi r_S}\exp(-ikr_S)\,q, \quad r_S = \sqrt{(x - x_S)^2 + (y - x_S)^2 + d_S^2}$$

The surface is divided into N_S surface elements and each surface element represents the origin of a secondary wave. The source strength is determined from the particle velocity just before hitting the surface and only the normal component of the particle velocity is considered (hence the cosine).

$$u_S = \frac{p_S}{\rho_0 c}\cos(\upsilon_S), \quad \cos(\upsilon_S) = \frac{r_S^2 + d_S^2 - \left((x - x_S)^2 + (y - y_S)^2\right)}{2 r_S d_S} = \frac{d_S}{r_S}$$

The volume velocity is calculated from the particle velocity multiplied by the surface element area $\Delta S = \Delta x \Delta y$.

$$\Delta q_S = u_S \Delta S = \frac{d_S}{r_S}\Delta x \Delta y \frac{p_S}{\rho_0 c}$$

The secondary wave is a point source. The directivity of the wavelet along the normal is included through the cosine function, which is set to unity if not used. This is a "switch" to enable diffusion according to Lambert's law.

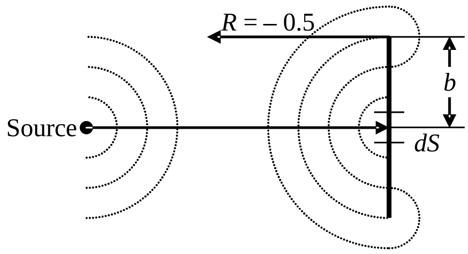
$$\Delta p_2 = \frac{ik\rho_0 c}{4\pi r_2}\exp(-ikr_2)\cos(\upsilon_R)\Delta q_S, \quad r_2 = \sqrt{(x-x_R)^2+(y-y_R)^2+d_R^2}, \quad \cos(\upsilon_R) = \frac{d_R}{r_2}$$

Insertion of Δq_S and p_S results in an expression for the scattered signal.

$$\Delta p_2 = \frac{ik\rho_0 c}{4\pi r_2}\exp(-ikr_2)\frac{d_R}{r_2}\frac{d_S}{r_S}\Delta x\Delta y\frac{ik}{4\pi r_S}\exp(-ikr_S)\,q, \quad \text{where} \quad \frac{d_R}{r_2}\ \text{is optional}$$

3.1.3. Diffracted sound

The sound pressure p_3 at the receiver is due to the wave front propagating toward the edge of the panel where the pressure suddenly drops and the wave front becomes divided into a reflected wave with the reflection coefficient[3] R = –0.5 thus leaving a transmitted wave with the transmission coefficient T = 0.5 radiating into the shadow zone behind the panel (but this is not considered here). The reflection coefficient is related to the angle to the receiver and is frequency dependent; however these complications will be ignored as the contribution from the edge diffraction is small compared to the scattered signal.



The pressure is suddenly reduced when the wave front passes the edge leaving 2π space and entering 4π space. This cause reflection to occur with the reflection coefficient *R* = –0.5.

The sound propagates a total distance of b from the surface element to the edge and then r_3 from the edge to the receiver[4] and suffers attenuation and sign inversion due to the reflection coefficient.

The edge is divided into N_B equal segments located at (x_n, y_n) and each contributing $1/N_B$ to the total diffraction since each surface point is illuminating the full boundary, so the angle of incidence at the edge is not important since all the sound hits the boundary and gets diffracted. The 4π within the denominator is changed to 2π since radiation is into the half space in front of the panel and a cosine square function can be implemented to model zero radiation in direction of the edge while full level is obtained toward the centre of the panel[5].

[3] The reflection coefficient was determined through literature studies, theory and measurements using a thin baffle and is reported within my master project. The value was –0.50 according to Lord Rayleigh (Bews & Hawksford, 1986); a finite element analysis yielded –0.58 (Wright, 1996); my impedance study lead to –0.60; and monitored values using circular disks were –0.71 (Vanderkooy, 1991) and –0.77 according to myself. Best fit was found at –0.50 as the result of several measurements for the relation between a point source at the surface of the disk and the far field sound pressure using circular and rectangular shapes with different positions of the source [S-56 and S-62].

[4] This part can be debated since all references known to me use point sources arranged along the boundary and thus introducing a factor of $1/b$ for the point source at the surface of the panel and $1/r_3$ for radiation from the edge (see for instance Vanderkooy). However, this produces infinite sound pressure at the boundary, which is physically impossible. The proposed model corresponds to simple reflection and cannot increase the sound pressure ad infinitum.

$$\Delta p_3 = \frac{R}{N_B}\frac{ik\rho_0 c}{2\pi(b+r_3)}\exp(-ik(b+r_3))\cos^2\left(\frac{\upsilon_D}{2}\right)\Delta q_S,\qquad \begin{array}{c} b=\sqrt{(x-x_n)^2+(y-y_n)^2} \\ r_3=\sqrt{(x_n-x_R)^2+(y_n-y_R)^2+d_R^2} \\ \cos^2\left(\frac{\upsilon_D}{2}\right)=\frac{1+\cos(\upsilon_D)}{2},\quad \cos(\upsilon_D)=\frac{d_R}{r_3}\end{array}$$

Insertion of Δq_S and p_S results in the expression of the diffracted signal where 1 + d_R/r_3 is to be substituted by the factor 2 is the cosine squared relation is not used.

$$\Delta p_3 = \frac{R}{N_B}\frac{ik\rho_0 c}{4\pi(b+r_3)}\exp(-ik(b+r_3))\left(1+\frac{d_R}{r_3}\right)\frac{d_S}{r_S}\Delta x\Delta y\,\frac{ik}{4\pi r_S}\exp(-ikr_S)\,q$$

Superposition – The resultant sound pressure at the monitoring point becomes the sum of the individual contributions integrated over the surface.

$$p = p_1 + \sum_{N_S}\Delta p_2 + \sum_{N_B}\Delta p_3$$

A transfer function is defined through division with p_1 thus eliminating the relation to the source strength and the proportionality to frequency. The amplitude of the transfer function is shown using the logarithmic decibel scale. The normalisation produces unity transfer level (0 dB) when both scattering and diffraction have been removed from the analysis.

$$H_1 = \frac{p}{p_1}\quad\Rightarrow\quad A = 20\log_{10}\left(1+\sum_{N_S}\frac{\Delta p_2}{p_1}+\sum_{N_S,N_B}\frac{\Delta p_3}{p_1}\right)\text{dB}$$

The relative sound pressure due to scattering is labelled H_2 within the software. It is seen that the amplitude of the scattered sound is proportional to frequency through the angular wave number appearing within the nominator. The square of the distances r_2 and r_S is due to the cosine functions defining the angle between source and panel and between panel and receiver; and if the relation is not used the factor d_R is removed and r_2 is reduced to first order.

$$H_2 = \sum_{N_S}\frac{\Delta p_2}{p_1} = \sum_{N_S}\frac{\frac{ik\rho_0 c}{4\pi r_2}\exp(-ikr_2)\,\frac{d_R}{r_2}\,\frac{d_S}{r_S}\,\Delta x\Delta y\,\frac{ik}{4\pi r_S}\exp(-ikr_S)\,q}{\frac{ik\rho_0 c}{4\pi d_1}\exp(-ikd_1)\,q}$$

After reduction:

$$H_2 = \frac{d_1 d_S d_R \Delta x\Delta y}{4\pi}\sum_{N_S}\frac{ik}{r_2^2 r_S^2}\exp(-ik(r_S+r_2-d_1))$$

The relative sound pressure due to edge diffraction is labelled H_3 within the software and the expression is much like the one for scattering so the level of diffraction is proportional to frequency as well. The increased distance for the diffracted signal is visible through the term $b + r_3$.

[5] The far field amplitude of the diffracted component is zero along the direction of the panel and is expected descried through the cosine function.

$$H_3 = \sum_{N_B} \frac{\Delta p_3}{p_1} = \sum_{N_B} \frac{\dfrac{R}{N_B} \dfrac{ik\rho_0 c}{4\pi(b+r_3)} \exp(-ik(b+r_3)) \left(1+\dfrac{d_R}{r_3}\right) \dfrac{d_S}{r_S} \Delta x \Delta y \dfrac{ik}{4\pi r_S} \exp(-ikr_S)\, q}{\dfrac{ik\rho_0 c}{4\pi d_1} \exp(-ikd_1)\, q}$$

After reduction:

$$H_3 = \frac{Rd_1 d_S \Delta x \Delta y}{4\pi N_B} \sum_{N_B} \frac{ik}{(b+r_3) r_S^2} \exp(-ik(r_S + b + r_3 - d_1)) v, \quad \text{where} \quad \left\{ v = 1 + \frac{d_R}{r_3} \quad \text{or} \quad v = 2 \right\}$$

The number of points at the surface N_S and the number of points along the boundary N_B are defined by the selected resolution.

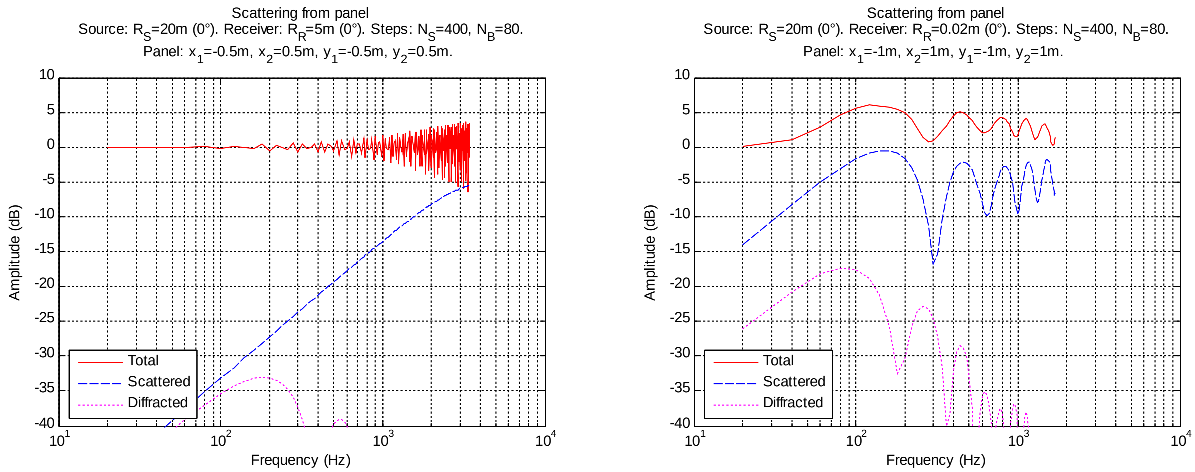
$$N_S = \frac{x_2 - x_1}{\Delta x} \frac{y_2 - y_1}{\Delta y}, \quad N_B = 2\left(\frac{x_2 - x_1}{\Delta x} + \frac{y_2 - y_1}{\Delta y} \right), \quad \begin{matrix} x_2 > x_1 \\ y_2 > y_1 \end{matrix}$$

The useful upper frequency is related to the size of the steps. Using half wave length as the limiting factor, the maximum usable frequency is given by the direction with the largest increment and for the selection $\Delta x = \Delta y = 100$ mm the theory becomes upward limited at approximately 1.7 kHz.

$$f_{\max} = \min\left(\frac{c}{2\Delta x}, \frac{c}{2\Delta y} \right)$$

3.1.4. Verifying the model

This section contains measurements for verification of the model.



Plots of the amplitude of the sound pressure with different setup for verification.

The far field amplitude of the scattered signal must be proportional to frequency (20 dB/decade).

d_S = 20 m, d_R = 5 m, panel 1 m by 1 m: Level –33.2 dB at 100 Hz and –13.5 dB at 1 kHz; change 19.7 dB.

The expected change was 20.0 dB.

The far field amplitude of the scattered signal must be inversely proportional to distance but this appear difficult to achieve so far field is indeed far away; d_S > 100 m.

1 kHz, d_S = 200 m was –33.2 dB at d_R = 50 m and –25.9 dB at d_R = 25 m; change 7.3 dB (6.0).
1 kHz, d_S = 100 m was –36.7 dB at d_R = 50 m and –27.2 dB at d_R = 25 m; change 9.5 dB (6.0).

1 kHz, d_S = 50 m was –27.2 dB at d_R = 20 m and –18.7 dB at d_R = 10 m; change 8.5 dB (6.0).

The expected change of 6.0 dB was not met.

The far field amplitude of the scattered signal must be proportional to area.

1 kHz, d_S = 50 m and d_R = 20 m was –15.5 dB at panel 2 m by 2 m; (0 dB reference).
1 kHz, d_S = 50 m and d_R = 20 m was –21.3 dB at panel 2 m by 1 m; change 5.8 dB (–6.0).
1 kHz, d_S = 50 m and d_R = 20 m was –27.2 dB at panel 1 m by 1 m; change 11.7 dB (–12.0).
1 kHz, d_S = 50 m and d_R = 20 m was –33.2 dB at panel 1 m by 0.5 m; change 17.7 dB (–18.0).
1 kHz, d_S = 50 m and d_R = 20 m was –39.2 dB at panel 0.5 m by 0.5 m; change 23.7 dB (–24.0).

The expected change is 6 dB for change in one direction, and this is met within ±0.3 dB.

With the source within the far field (simulated by 20 m distance) the pressure near the surface must approach two times the incident sound pressure (6.02 dB), which was met. Activating the cosine functions reduced the level to 0.6 dB so the cosine functions should not be used.

120 Hz, d_S = 20 m, d_R = 0.02 m, panel 1 m by 1m: level was 5.99 dB.

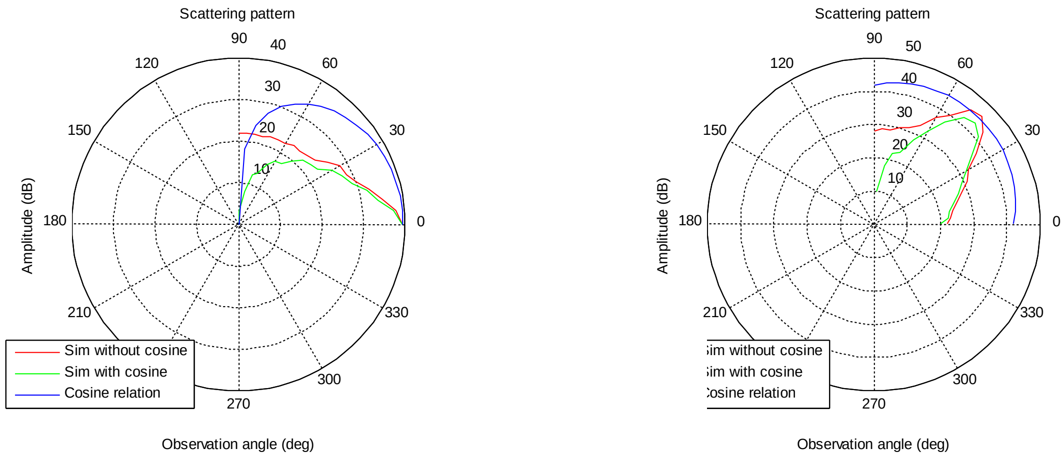
The expected level was 6.02 dB so this is met within ±0.03 dB.

The far field cut off frequency must be $f_{cut\ off} \approx 3cd/4b^2$ (see section 3.2) which is $f_{cut\ off} \approx$ 1.0 kHz for $d = d_S d_R /(d_S + d_R)$ = 4 m (d_S = 20 m and d_R = 5 m) b = 1 m.

d_S = 20 m, d_R = 5 m, panel 1 m x 1 m, peak at 1.02 kHz.

The expected level was 1.0 kHz and was met.

The amplitude should follow cosine to the observation angle but this was not the case (blue line). The inclusion of a cosine to emulate Lambert's cosine law reduced the level at large values of the observation angle but the theory is not proved (green line). The results shown below was simulated using the source at d_S = 20 m from the panel and the receiver at d_R = 5 m distance, with the different angles realised through $d_R = r_R \cos(\theta)$ and $x_R = r_R \sin(\theta)$. The panel was 2 m by 2 m and for the second measurements the source were located at $d_S = x_S$ = 14.14 m and the receiver at negative angle (although reported below in the first quadrant).



The relation is shown for the scattered signal versus the observation angle. To the left with the source and receiver along the normal and to the right with the source and receiver offset by ±45º, along the expected reflection paths.

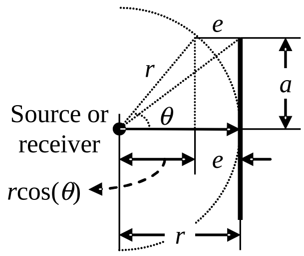
This result is interesting since it relates well to the intuitive expectation of sound scattering from a panel as a major lobe according to Snell's law with some spread. The width of the beam is around ±10° at –3 dB and seems to be slightly narrower at 45° of incidence.

The software did not pass all tests; the far field amplitude was only approximately inversely proportional to distance and the relation to angle was not as expected from the analytical expression shown within the introduction. Anyway, the major parts were verified.

All measurements within this report used the setup without enabling the cosine function.

3.2. Frequency limit

The signal being scattered from the panel is not in phase for all contributions with the exception of the situation with source and receiver within the far field. The signal near the edge of the panel is delayed due to the additional travelling time and destructive interference occurs when the signal has been delayed one half wave length. The frequency where the signal is delayed one quarter of the wave length marks the set on of cancellation since the delayed signal from the edge is 90º behind the signal from the centre.



The spherical wave form reaches the centre before the edge so the signal from the outer range is delayed with respect to the scattering from the centre of the panel.

The delay will be calculated for the distance *e* from the edge to the circle periphery. The angle θ to the edge of the panel is slightly less than the angle to the point where *e* touches the periphery but the approximation becomes valid for sufficiently large distance and the same assumption can be used to simplify the sine by using the first term of the Taylor series expansion [RW-197].

$$\sin(\theta) = \frac{a}{r} \quad \Rightarrow \quad \theta \approx \frac{a}{r}$$

The length of *e* is determined from the projection $r\cos(\theta) + e = r$ and the cosine is approximated by the first two terms of the Taylor series expansion [RW-198].

$$e \approx r - r\cos(\theta) \approx \left[1 - \cos\left(\frac{a}{r}\right)\right] r \quad \Rightarrow \quad e \approx \left[1 - \left(1 - \frac{1}{2}\left(\frac{a}{r}\right)^2 + \ldots\right)\right] r \approx \frac{1}{2}\left(\frac{a}{r}\right)^2 r \quad \Rightarrow \quad e \approx \frac{a^2}{2r}$$

The cut off frequency will be calculated for the source and the receiver together as the sum of the distances equating $\lambda/4$.

$$e_S + e_R = \frac{\lambda}{4} = \frac{c}{4 f_{\text{cut off}}} \quad \Rightarrow \quad f_{\text{cut off}} = \frac{c}{4(e_S + e_R)}$$

Using the expression for the distance *e* gives:

$$f_{\text{cut off}} = \frac{c}{4\left(\frac{a^2}{2r_S} + \frac{a^2}{2r_R}\right)} = \frac{c}{4a^2} \frac{1}{\frac{1}{2r_S} + \frac{1}{2r_R}} = \frac{c}{4a^2} \frac{1}{\frac{r_R + r_S}{2r_S r_R}} = \frac{c}{4a^2} \frac{2r_S r_R}{r_R + r_S}$$

The term $2r_Sr_R/(r_S + r_R)$ is the harmonic mean of two components and appears to be used within acoustics as the mean of the distance to source and receiver.

A formula originating from Rindel due to a study of Fresnel diffraction from small panels is quoted by Cox and D'Antonio and shown below; it uses the object radius a, the distance to source and receiver (r and r_0) and the angle of the incident wave [CA-332]. The expression reduces to the one derived above for $\psi = 45°$, although this is not the assumption for the derivation of the expression for the circular panel.

$$f_{-3\mathrm{dB}} \approx \frac{\mathrm{c}r^*}{8\mathrm{a}^2\cos^2(\psi)}, \quad r^* = \frac{2rr_0}{r + r_0}$$

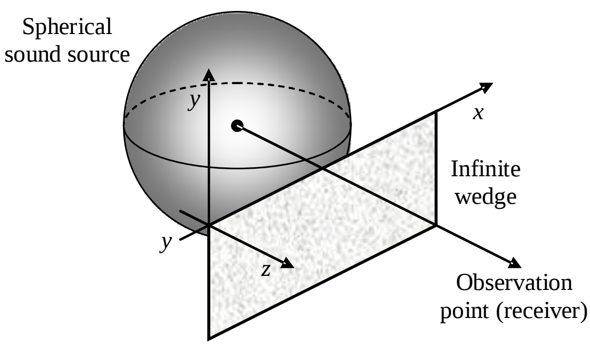
The cut off frequency is shown below for a circular panel with radius a and the distances r_S to the source and r_R to the receiver. For a square panel, the equation can be written using the side length b by equating areas: $\pi a^2 = b^2$. Kuttruff reports the same formula for the circular panel, and includes a formula for a long and narrow panel, which is reproduced below using h for the height or horizontal width of the panel.

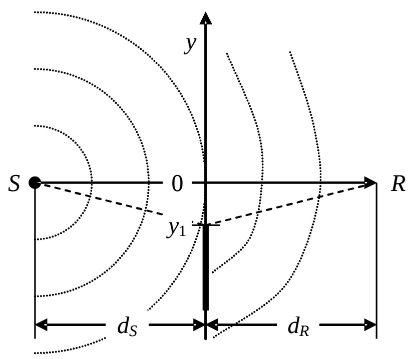
$$\left.\begin{array}{ll} \text{Circular panel:} & f_{\text{cut off}} \approx \dfrac{c\hat{r}}{4a^2} \\ \text{Square panel:} & f_{\text{cut off}} \approx \dfrac{\pi\, c\hat{r}}{4b^2} \\ \text{Narrow panel:} & f_{\text{cut off}} \approx \dfrac{c\hat{r}}{2h^2} \end{array}\right\} \quad \hat{r} = \frac{2r_S r_R}{r_S + r_R}$$

3.3. Fresnel near-field approximation

Whenever a propagating wave is disturbed by a change within the impedance of the medium a set of secondary waves (*wavelets*) are generated, according to the principle of Huygens (1629-1695) where each air particle is the source of spherical waves. Fresnel (1788-1827) introduced an oblique factor attenuating the diffracted waves according to the direction in order to suppress the backward-travelling wave and this was later placed on more rigorous basis by Kirchhoff (1824-1887).

A baffle is used to interrupt the path from a noise source to a destination; examples are the noise barrier close to a high way or two persons in conversation over a screen at eye height. As a model the semi-infinite baffle is used, which blocks sound propagation below a certain height. Propagation of the sound after passage of the baffle is calculated using wavelets and two theories are available; the approximations due to Fraunhofer for the far field [S-98] and Fresnel for the near field [S-116]. The theories are outlined in my Master project and shall not be repeated here; only selected results will be quoted.





A spherical wave front is disturbed by a semi infinite baffle and the reflections from the edge causes interference at the monitoring point.

The setup to be referenced is shown above where sound propagation below the edge of the baffle is eliminated. The straight line of sight from the centre of the point source to the monitoring point is the zero reference. The source is located at distance d_S to the left of the baffle with the receiver at distance d_R to the right of the baffle; the edge of the baffle is located at y_1 with the positive direction upward so positive values correspond to blocking the direct sound [S-128].

The *Fresnel length* is defined [S-119]. The symmetry of the expression illustrates the principle of reciprocity according to which the source and receiver can be interchanged without impact upon the result. The Fresnel length is less than the shortest distance; and for large distance from the source to the baffle we have $L \approx d_R$ and similar for large distance to the receiver.

$$L = \frac{d_S d_r}{d_S + d_R}$$

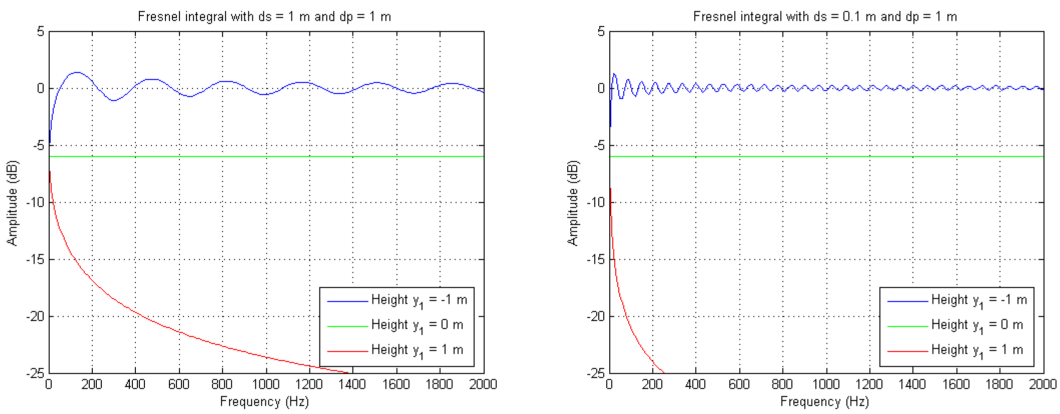
A dimensionless frequency-distance variable v is also defined, which is related to frequency through the angular wave number $k = 2\pi f/c$ and to distance through the Fresnel length. [S-120].

$$v = \sqrt{\frac{2}{L\lambda}} y_1 = \sqrt{\frac{kL}{\pi}} \frac{y_1}{L}$$

The frequency response of the signal at the monitoring point is shown below using the Fresnel cosine and sine integrals $C(v)$ and $S(v)$, which are introduced in the next section [S-128].

$$\boxed{H \propto \tfrac{1}{2} - C(v) - i\left(\tfrac{1}{2} - S(v)\right)}$$

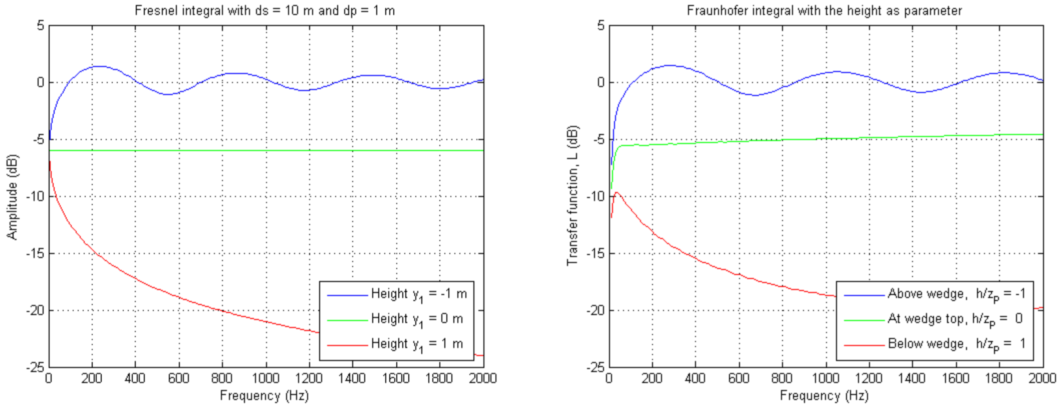
Plots of the resulting transfer function are shown below using three values of d_S with constant d_R and three values of y_1. The blue curve is with the edge below the line-of-sight (y_1 = –1 m) so the direct signal is not blocked and the diffracted signal causes interference shown as the oscillations. The green curve is with the edge positioned just at the line-of-sight (y_1 = 0) and results in 6 dB level reduction but without interference. The red curve is with the edge obstructing the direct signal from the source (y_1 = 1 m) so only the diffracted signal is monitored and there is no interference.



Transfer function with source 1 m from baffle (left) or 0.1 m (right) and the observer 1 m from baffle measured at three different edge heights. The parameter d_P corresponds to d_R used within this document.

The plots below compare the results to the Fraunhofer theory with a setup using $z_P = d_R$ and $h = y_1$ and shows that the theories are largely equivalent in the far field. The unobstructed level is –5 dB at zero frequency, which is unrealistic since it corresponds to static pressure difference so this is a simulation artefact. The first peak is located as shown below and this may represent the lower frequency limit of the model.

$$f_{peak} \approx \left(230\,\mathrm{Hz/m}\right) \cdot L$$



Transfer function with source 10 m from baffle (left) and compared to the Fraunhofer far-field approximation for an observer 1 m from the baffle (right).

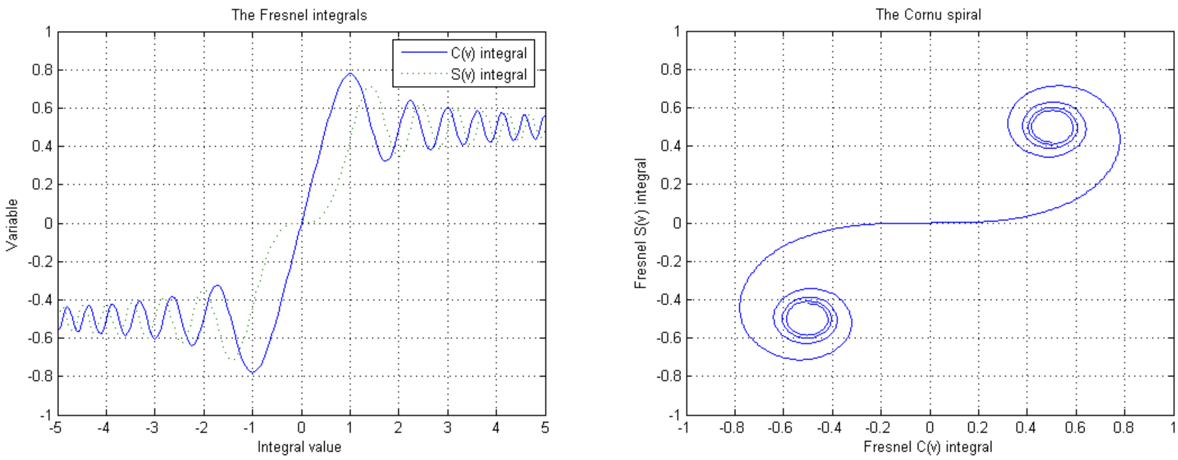
The phase of the diffracted signal is changed from the incident wave, which can be seen from the curve for $d_S = d_R = 1$ m with $y_1 = -1$ m. The path from source via the edge to the monitoring point introduces an additional delay for the diffracted signal of 1.41 m, which is equivalent to half wavelength at $f = c/2\lambda = 120$ Hz where a peak is observed thus indicating phase inversion of the diffracted signal. However, the next dip is at 290 Hz where 240 Hz was expected (full wave length).

3.4. Fresnel cosine and sine integrals

The Fresnel cosine and sine integrals are defined as:

$$C(v) = \int_0^v \cos\left(\frac{\pi}{2}\gamma^2\right) d\gamma \quad \wedge \quad S(v) = \int_0^v \sin\left(\frac{\pi}{2}\gamma^2\right) d\gamma \quad \xrightarrow[|v|\to\infty]{} \quad \begin{cases} \frac{1}{2}, & v \to \infty \\ -\frac{1}{2}, & v \to -\infty \end{cases}$$

The cosine and sine integrals are plotted below, showing that the asymptotic value is approached only slowly for large values of the integration value v and that the integrals are oscillating.



The Fresnel cosine and sine integrals are shown to the left showing the oscillations, and the Cornu spiral is shown to the right using the cosine integral as abscise and the sine integral as ordinate.

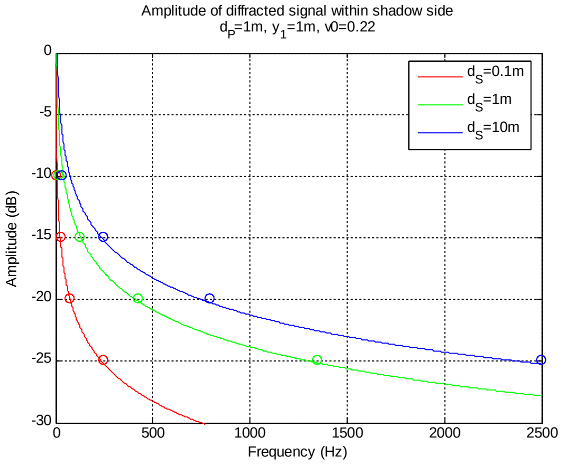
An alternative display is plotting $S(v)$ versus $C(v)$ with v as parameter, which is known as the Cornu spiral. Integration from 0 to v corresponds to the straight line from (0, 0) to the point on the curve corresponding to the value v. For v moving from zero through infinity the length increases toward a maximum value (of approximately $0.78 + i0.45$) and decreases toward $0.5 + i0.5$ through an infinite number of oscillation periods.

3.5. Amplitude of diffracted signal

The attenuation of the diffracted signal within the shadow zone is correlated to v for values not too close to zero. This is shown below where the amplitude of the diffracted signal A_D is given by the Fresnel variable. The constant v_0 is found experimentally using the simulation results; for example, the curve for $d_S = d_R = 1$ m corresponds to $L = 0.5$ m and attenuation is $A_D = -15$ dB at $f = 130$ Hz so $v = 1.23$ according to the definition and $v_0 = 0.219$ with very similar values for the other curves.

$$A_D = 20\log_{10}\left(\frac{v_0}{v}\right)\text{dB} \quad \Rightarrow \quad v_0 = v\exp_{10}\left(\frac{A_D}{20\,\text{dB}}\right) \approx 0.22 \quad \text{at} \quad y_1 = 1\text{m}$$

For $v = v_0$ the estimation gives 0 dB, where –6 dB is the correct value, and the result is garbage at lower values, so a lower limit of $v > 0.5$ is suggested. The relation to y_1 has not been investigated and since the result obviously is incorrect near zero the parameter must at least be non-zero, with a proposed limit of $y_1 > L$. Assuming $v > 0.5$ and $y_1 = L$ the requirement becomes $f > c/8L$ and the relation can thus be expected useful at frequencies above 85 Hz for $L = 0.5$ m.



The amplitude of the diffracted signal within the shadow side with circles marking the values used to determine the parameters.

The estimated amplitude A_D can be converted into sound pressure p_D as shown below where p_0 is the level at low frequencies, which must approach the sound pressure of the incident wave at low frequencies.

$$p_D = \frac{v_0}{v}p_0 = \frac{v_0}{\sqrt{\frac{kL}{\pi}\frac{y_1}{L}}}p_0 = \sqrt{\frac{c}{2fL}}\frac{v_0 L}{y_1}p_0 \quad \Rightarrow \quad \boxed{p_D \approx p_0\sqrt{\frac{c}{40fL}}\frac{L}{y_1} \quad \text{where} \quad \begin{array}{c} y_1 > L \\ f > c/8L \end{array}}$$

4. MATLAB code

4.1. Level of diffracted sound

```
% DiffractedLevel.m
clear
dP=1;           % Distance to monitor (m).
y1=1;           % Baffel height above line of sight (m).
v0=0.22;        % Constant.
c=343;          % Speed of sound (m/s).
f=10:10:2500;   % Frequency (Hz).
k=2*pi*f/c;     % Angular wave number (m-1).
dS=[0.1 1 10];  % Distance to source (m).
C=['r' 'g' 'b'];
for n=1:3
    L=dS(n)*dP/(dS(n)+dP);
    v=sqrt(k*L/pi)*y1/L;
    AD=20*log10(v0./v);
    plot(f,AD,C(n))
    hold on
end
title({'Amplitude of diffracted signal within shadow side', ...
       ['d_P=1m, y_1=1m, v0=' num2str(v0)]})
xlabel('Frequency (Hz)')
ylabel('Amplitude (dB)')
legend(['d_S=' num2str(dS(1)) 'm'],['d_S=' num2str(dS(2)) 'm'],['d_S=' num2str(dS(3)) 'm'])
grid on
Ax=[-10 -15 -20  -25];  % Amplitude (dB).
f0=[ 10  30  80  250];  % Frequency for dS=0.1m (Hz).
f1=[ 20 130 430 1350];  % Frequency for dS=1m (Hz).
f2=[ 35 250 800 2500];  % Frequency for dS=10m (Hz).
plot(f0,Ax,'or', f1,Ax,'og', f2,Ax,'ob')
hold off
axis([0 2500 -30 0])
```

4.2. Scattering from panel

```
% ScatteringFromPanel.m
clear
% === SOURCE ===
ds=20.0;            % Distance from source to panel (m).
xs= 0.0;            % Source offset (m).
ys= 0.0;
% === RECEIVER ===
dr= 0.4;            % Distance from receiver to panel (m).
xr= 0.0;            % Receiver offset (m).
yr= 0.0;
% === PANEL ===
x1=-1.000;          % Panel dimensions (m).
x2= 1.000;
y1=-1.000;
y2= 1.000;
% === SIMULATOR ===
R=-0.5;             % Edge reflection coefficient (0 = no diffraction).
fmin=20;            % Start and step frequency (Hz).
N=20;               % Number of steps in x and y directions.
usecosine = 0;      % Switch the Fresnel cosines in (1) or out (0, default).
c=343;              % Speed of sound (m/s).
% === CALCULATION ===
Rs=sqrt(ds^2+xs^2+ys^2);% Resultant distance from source to panel (m).
Rr=sqrt(dr^2+xr^2+yr^2);% Resultant distance from receiver to panel (m).
Vs=(180/pi)*acos((ds^2+Rs^2-xs^2-ys^2)/(2*ds*Rs));  % Source angle (rad).
Vr=(180/pi)*acos((dr^2+Rr^2-xr^2-yr^2)/(2*dr*Rr));  % Receiver angle (rad).
sr=ds-dr+xs-xr+ys-yr;   % Distance between source and receiver (m).
if (abs(sr)<ds/100), disp('Source and receiver are too close'), break, end
dx=(x2-x1)/N;       % Step size for integration (m).
dy=(y2-y1)/N;
fmax=round(min(c/(2*dx),c/(2*dy)));
f=fmin:fmin:fmax;   % Frequency axis (Hz).
ik=i*2*pi*f/c;      % Angular wave number (m-1).
```

```
H1=1;                % Switch to include the direct signal (1) or not (0).
H2=0;                % Prepare addition of the reflection terms.
H3=0;                % Prepare addition of the diffraction terms.
NS=(x2-x1)*(y2-y1)/(dx*dy);     % Number of surface elements.
NB=2*((x2-x1)/dx+(y2-y1)/dy);   % Number of boundary segments.
disp([num2str(NS*NB*length(f)) ' steps from ' num2str(fmin) ' Hz to ' num2str(fmax) ' Hz'])
for x=(x1+dx/2):dx:(x2-dx/2)    % Integrate through the surface.
    for y=(y1+dy/2):dy:(y2-dy/2)
        d1=sqrt((xs-xr)^2+(ys-yr)^2+(ds-dr)^2);
        rs=sqrt((x-xs)^2+(y-ys)^2+ds^2);
        r2=sqrt((x-xr)^2+(y-yr)^2+dr^2);
        if (usecosine==1) H2=H2+(ik/(rs^2*r2^2)).*exp(-ik*(r2+rs-d1));
        else              H2=H2+(ik/(rs^2*r2)).*exp(-ik*(r2+rs-d1)); end
        for xn=x1:dx:x2-dx      % Integrate along the boundary.
            yn=y1;
            b=sqrt((x-xn)^2+(y-yn)^2);
            r3=sqrt((xn-xr)^2+(yn-yr)^2+dr^2);
            if (usecosine==1) H3=H3+(ik*(1+dr/r3)/((b+r3)*rs^2)).*exp(-ik*(rs+b+r3-d1));
            else              H3=H3+(ik*2/((b+r3)*rs^2)).*exp(-ik*(rs+b+r3-d1)); end
        end
        for yn=y1:dy:y2-dy
            xn=x2;
            b=sqrt((x-xn)^2+(y-yn)^2);
            r3=sqrt((xn-xr)^2+(yn-yr)^2+dr^2);
            if (usecosine==1) H3=H3+(ik*(1+dr/r3)/((b+r3)*rs^2)).*exp(-ik*(rs+b+r3-d1));
            else              H3=H3+(ik*2/((b+r3)*rs^2)).*exp(-ik*(rs+b+r3-d1)); end
        end
        for xn=x2:-dx:x1+dx
            yn=y2;
            b=sqrt((x-xn)^2+(y-yn)^2);
            r3=sqrt((xn-xr)^2+(yn-yr)^2+dr^2);
            if (usecosine==1) H3=H3+(ik*(1+dr/r3)/((b+r3)*rs^2)).*exp(-ik*(rs+b+r3-d1));
            else              H3=H3+(ik*2/((b+r3)*rs^2)).*exp(-ik*(rs+b+r3-d1)); end
        end
        for yn=y2:-dy:y1+dy
            xn=x1;
            b=sqrt((x-xn)^2+(y-yn)^2);
            r3=sqrt((xn-xr)^2+(yn-yr)^2+dr^2);
            if (usecosine==1) H3=H3+(ik*(1+dr/r3)/((b+r3)*rs^2)).*exp(-ik*(rs+b+r3-d1));
            else              H3=H3+(ik*2/((b+r3)*rs^2)).*exp(-ik*(rs+b+r3-d1)); end
        end
    end
end
if (usecosine==1)
    H2=H2*d1*ds*dr*dx*dy/(4*pi);      % Scattered sound  (S = surface).
    H3=H3*R*d1*ds*dx*dy/(4*pi*NB);   % Diffracted sound (B = boundary).
else
    H2=H2*d1*ds*dx*dy/(4*pi);         % Scattered sound  (S = surface).
    H3=H3*R*d1*ds*dx*dy/(4*pi*NB);      % Diffracted sound (B = boundary).
end
H1=H1+H2+H3;                        % Total sound.
semilogx(f,20*log10(abs(H1)),'-r',...
         f,20*log10(abs(H2)),'--b',...
         f,20*log10(abs(H3)),':m')
title({'Scattering from panel',...
       ['Source: R_S=' num2str(round(1000*Rs)/1000) 'm (' num2str(round(Vs)) '°). ' ...
        'Receiver: R_R=' num2str(round(1000*Rr)/1000) 'm (' num2str(round(Vr)) '°). ' ...
        'Steps: N_S=' num2str(NS) ', N_B=' num2str(NB) '.'], ...
       ['Panel: x_1=' num2str(x1) 'm, x_2=' num2str(x2) 'm, ' ...
        'y_1=' num2str(y1) 'm, y_2=' num2str(y2) 'm. ' ...
        (usecosine==1)*'Cos(\theta) active']})
xlabel('Frequency (Hz)')
ylabel('Amplitude (dB)')
legend('Total','Scattered','Diffracted','Location','SouthWest')
axis([10 10000 -40 10])
grid on
```

4.3. Polar plot of scattering

```
% DiffuserPlot.m
clear
format compact
N=11;       % Number of wells within the diffuser (prime > 2).
```

```
r= 2;       % Used for primitive root of N.
m= 1;       % Offset for QRD.
P= 1;       % Number of periods.
w=0.032;    % Well depth (m).
%f=5360;     % Critical frequency (Hz).
f=2000;     % Frequency (Hz).
c=343;      % Speed of sound (m/s).
theta=-pi/2:0.01:pi/2;
disp('...')
disp(['QRD: N=' num2str(N) ', m=' num2str(m)])
Hn=0;
smax=0;
for n=1:N*P
    %m=round(n/(N*P));       % Experiment with asymmetry.
    sn=mod(((n-1)^2+m),N);  % QRD: Sequence term number n.
    %sn=mod(r^(n-1),N);      % PRS: Sequence term number n.
    if (sn>smax) smax=sn; end
    dn=w*sn;                % Well depth number n (m).
    if (dn>0) Rn=-(1+i*cot(2*pi*f*dn/c))/(1-i*cot(2*pi*f*dn/c));
    else      disp('Well length of zero'), Rn=1; end
    %Rn=1;                  % Reflection coefficient for plane surface.
    Hn=Hn+Rn*exp(-i*2*pi*f*n*w*sin(theta)/c);
end
disp(['Well depth: smax=' num2str(smax) ' (' num2str(100*smax/(N-1)) '% of N-1)'])
Hn=max(0,20*log10(abs(Hn)));
polar(theta,Hn,'-r')
title({'Schröder diffuser design', ...
      ['N=' num2str(N) ', m=' num2str(m) ', w=' num2str(w) 'm' ...
      ', P=' num2str(P) ', f=' num2str(f) 'Hz']})
xlabel('Observation angle (deg), Amplitude (dB)')
```

4.4. Plot of the scattering direction

```
% PlotOfCosineRelation.m

v=0:5:90;
vp=pi*v/180;
% Normal incidence (source at 0 deg).
% a0: Without cosine weighting, a1: With cosine weighting.
a0=50+[-10.5 -12 -15 -17.5 -20 -21.5 -22 -24 -26 -26.5 -27 -27 -27.5 -27.5 -27.5 -28 -28 -28
-28];
a1=50+[-10.5 -12.5 -16 -18 -21 -22.5 -24 -27 -27.5 -28 -30 -32 -32.5 -34 -37 -37.5 -42 -47
-50];
a2=max(0,50+20*log10(cos(pi*v/180))-10.5);
% Oblique incidence (source at 45 deg).
% a0: Without cosine weighting, a1: With cosine weighting.
%a0=50+[-28 -27 -26 -24 -22 -19 -17 -12.5 -7 -4 -5 -10 -13 -17 -19 -20 -21 -21 -22];
%a1=50+[-30 -27.5 -27 -25 -23 -21 -18 -14 -9 -7 -8 -12.5 -17.5 -22 -27 -28 -32 -40 -inf];
%a2=max(0,50+20*log10(cos(pi*v/180-pi/4))-5);
polar(vp,a0,'-r')
hold on
polar(vp,a1,'-g')
polar(vp,a2,'-b')
hold off
title('Scattering pattern')
xlabel('Observation angle (deg)')
ylabel('Amplitude (dB)')
legend('Sim without cosine','Sim with cosine','Cosine relation','Location','SouthWest')
grid on
```

5. References

The following references are used within the reports.

B: Leo Beranek *Acoustics*, Reprinted 1996, Acoustical Society of America.

CA: Trevor Cox and Peter D'Antonio *Acoustic Absorbers and Diffusers*, 2nd Ed. 2009, Spon Press

K: Heinrich Kuttruff *Room Acoustics*, 5th Ed. 2009, Spon Press.

L: W. Marshall Leach *Introduction to Electroacoustics*, 3rd Ed. 2003, Kendall/Hunt.

RW: Lennart Råde and Bertil Westergreen *Mathematics Handbook*, 5th Ed. 2004, Studentlitteratur.

R1: Tore Skogberg *Report 1 – Reverberation*, External project, 2010, DTU.

R2: Tore Skogberg *Report 2 – Absorber*, External project, 2010, DTU.

S: Tore Skogberg *Loudspeaker Cabinet Diffraction*, Master project, 2006, DTU.

Although the Internet cannot be used as a scientific reference, the *Uniform Resource Locator* (URL), i.e. the Internet "address" will be used for lightweight references.